



# MEASUREMENT OF THERMAL EXPANSION COEFFICIENT USING STRAIN GAGES

## Tech Note TN-513-1

The thermal expansion coefficient is a very basic physical property which can be of considerable importance in mechanical and structural design applications of a material. Although there are many published tabulations of expansion coefficients for the common metals and standard alloys, the need occasionally arises to measure this property for a specific material over a particular temperature range. In some cases (e.g., new or special alloys, composites, etc.), there is apt to be no published data whatsoever on expansion coefficients. In others, data may exist (and eventually be found), but may encompass the wrong temperature range, apply to somewhat different material, or be otherwise unsuited to the application.

Historically, the classical means for measuring expansion coefficients has been the “dilatometer”. In this type of instrument, the difference in expansion between a rod made from the test material and a matching length of quartz or vitreous silica is compared<sup>1,2</sup>. Their differential expansion is measured with a sensitive dial indicator, or with an electrical displacement transducer. When necessary, the expansion properties of the quartz or silica can be calibrated against the accurately known expansion of pure platinum or copper. The instrument is normally inserted in a special tubular furnace or liquid bath to obtain the required temperatures. Making measurements with the dilatometer is a delicate, demanding task, however, and is better suited to the materials science laboratory than to the typical experimental stress analysis facility. This Tech Note provides an alternate method for easily and quite accurately measuring the expansion coefficient of a test material with respect to that of any reference material having known expansion characteristics.

The technique described here uses two well-matched strain gages, with one bonded to a specimen of the reference material, and the second to a specimen of the test material. The specimens can be of any size or shape compatible with the available equipment for heating and refrigeration (but specimens of uniform cross section will minimize potential problems with temperature gradients). Under stress-free conditions, the differential output between the gages on the two specimens, at any common temperature, is equal to the differential unit expansion (in/in, or *m/m*). Aside from the basic simplicity and relative ease of making thermal expansion measurements by this method, it has the distinct advantage of requiring no specialized instruments beyond those normally found in a stress analysis laboratory. This technique can also be applied to the otherwise difficult task

of determining directional expansion coefficients of materials with anisotropic thermal properties.

Because typical expansion coefficients are measured in terms of a few parts per million, close attention to procedural detail is required with any measurement method to obtain accurate results; and the strain gage method is not an exception to the rule. This Tech Note has been prepared as an aid to the gage user in utilizing the full precision of the modern foil strain gage for determining expansion coefficients. Given in the first of the following sections is an explanation of the technical principles underlying the method. The next section describes, in some detail, the strain-gage-related materials and procedures in making the measurement. Basically, the latter consists of essentially the same techniques required for any high-precision strain measurement in a variable thermal environment. Suggested refinements for achieving maximum accuracy are then given in the following section; after which, the principal limitations of the method are described.

### Principle of The Measurement Method

When a resistance strain gage is installed on a stress-free specimen of any test material, and the temperature of the material is changed, the output of the gage changes correspondingly. This effect, present in all resistance strain gages, was formerly referred to as “temperature-induced apparent strain”, but is currently defined as **thermal output**<sup>3</sup>. It is caused by a combination of two factors. To begin with, in common with the behavior of most conductors, the resistivity of the grid alloy changes with temperature. An additional resistance change occurs because the thermal expansion coefficient of the grid alloy is usually different from that of the test material to which it is bonded. Thus, with temperature change, the grid is mechanically strained by an amount equal to the difference in expansion coefficients. Since the gage grid is made from a strain-sensitive alloy, it produces a resistance change proportional to the thermally induced strain. The thermal output of the gage is due to the combined resistance changes from both sources. The net resistance change can be expressed as the sum of resistivity and differential expansion effects as follows:

$$\frac{\Delta R}{R} = [\beta_G + (\alpha_s - \alpha_G) F_G] \Delta T \quad (1)$$

### Measurement of Thermal Expansion Coefficient Using Strain Gages

where:

$\Delta R/R$  = unit resistance change

$\beta_G$  = thermal coefficient of resistivity of grid material

$\alpha_S - \alpha_G$  = difference in thermal expansion coefficients between specimen and grid, respectively

$F_G$  = gage factor of the strain gage

$\Delta T$  = temperature change from arbitrary initial reference temperature

The indicated strain due to a resistance change in the gage is:

$$\varepsilon_i = \frac{\Delta R / R}{F_I} \quad (2)$$

where:  $F_I$  = instrument gage factor setting

Then, the thermal output in strain units can be expressed as:

$$\varepsilon_{TIO(G/S)} = \frac{[\beta_G + (\alpha_S - \alpha_G)F_G]\Delta T}{F_I} \quad (3)$$

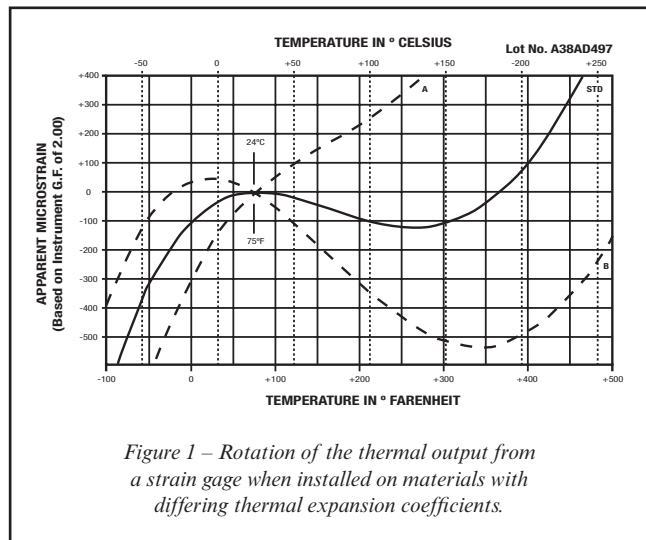
where:

$\varepsilon_{TIO(G/S)}$  = thermal output for grid alloy  $G$  on specimen material  $S$

Or, in the usual case, with the instrument gage factor set equal to that of the strain gage, so that  $F_I = F_G$ ,

$$\varepsilon_{TIO(G/S)} = \left[ \frac{\beta_G}{F_G} + (\alpha_S - \alpha_G) \right] \Delta T \quad (4)$$

It should not be assumed from the form of Equation (4) that the thermal output is linear with temperature, since all of the coefficients within the brackets are themselves functions of temperature. As an example, typical thermal output characteristics for a Vishay Micro-Measurements A-alloy gage (self-temperature-compensated constantan grid), bonded to steel, are represented by the solid curve in Figure 1. The lot of foil identified in the upper right corner of the graph was specially processed to minimize the thermal output over the temperature range from about  $-50^\circ$  to  $+300^\circ\text{F}$  [ $-45^\circ$  to  $+150^\circ\text{C}$ ]. Strain gages fabricated from this



lot of foil are intended for use only on material such as steel with a coefficient of expansion of approximately  $6 \times 10^{-6}/^\circ\text{F}$  [ $11 \times 10^{-6}/^\circ\text{C}$ ]. If the gages are installed on some other material with a different coefficient of expansion, the result is to effectively rotate the curve in Figure 1 about its reference point at  $+75^\circ\text{F}$  [ $+24^\circ\text{C}$ ]. Installation on a material with a higher coefficient of expansion than steel will rotate the curve counterclockwise, while a material with a lower expansion coefficient than steel will cause clockwise rotation. For example, the broken curve labeled **A** in the figure illustrates the general effect of installing a gage from the subject lot on a beryllium alloy having an expansion coefficient of about  $9 \times 10^{-6}/^\circ\text{F}$  [ $16 \times 10^{-6}/^\circ\text{C}$ ]. Similarly, if a gage from this lot were bonded to a titanium alloy with a somewhat lower expansion coefficient than steel, the thermal output would be shifted in the manner of the broken curve labeled **B**.

The principle of measuring expansion coefficients with strain gages then becomes evident from Figure 1, since the rotation from one thermal output curve to the other is due only to the difference in thermal expansion properties between the materials represented by the two curves. An algebraic demonstration of the principle can be obtained by rewriting Equation (4) twice; once for the gage installed on a specimen of the test material of unknown expansion coefficient  $\alpha_S$ , and again for the same type of gage installed on a standard reference material with a known expansion coefficient  $\alpha_R$ :

$$\varepsilon_{TIO(G/S)} = \left[ \frac{\beta_G}{F_G} + (\alpha_S - \alpha_G) \right] \Delta T \quad (5a)$$

$$\varepsilon_{TIO(G/R)} = \left[ \frac{\beta_G}{F_G} + (\alpha_R - \alpha_G) \right] \Delta T \quad (5b)$$

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Subtracting Equation (5b) from (5a), and rearranging,

$$\alpha_S - \alpha_R = \frac{(\epsilon_{TIO(G/S)} - \epsilon_{TIO(G/R)})}{\Delta T} \quad (6)$$

Thus, the difference in expansion coefficients, referred to a particular temperature range, is equal to the unit difference in thermal output for the same change in temperature. Although this technique for measuring expansion coefficients is widely applicable, and often the most practical approach, there is relatively little information about it in the technical literature. Representative applications are described in the bibliography to this Tech Note<sup>4,5</sup>.

### Measurement Procedures

#### Reference Material

Selection of the material to be used as a reference standard is naturally an important factor in the accuracy of the method, as it is for any other form of differential dilatometry. In principle, the reference material could be any substance for which the expansion properties are accurately known over the temperature range of interest. In practice, however, it is often advantageous to select a material with expansion properties as close to zero as possible. Doing this will provide an output signal that closely corresponds to the “absolute” expansion coefficient of the test material, and permits a more straightforward test procedure. The thermal expansion of the reference material should also be highly repeatable, and stable with time at any constant temperature. In addition, the elastic modulus of the material should be great enough that mechanical reinforcement by the strain gage is negligible.

\* Also available from Vishay Micro-Measurements as Part No. TSB-1. See Appendix for specimen dimensions.

An excellent reference material with these and the other desirable properties is ULE™ Titanium Silicate Code 7971, available from Corning Glass Company, Corning, NY 14831.\* As illustrated in Figure 2, this special glass has an extremely low thermal expansion coefficient, particularly over the temperature range from about -50° to +350°F [-45° to +175°C)]. It should be noted, however, that the material has a low coefficient of thermal conductivity, making it slow to reach thermal equilibrium. For optimum results, a dwell time of at least 45 minutes should be used at each new temperature point before taking data. Another potential disadvantage of titanium silicate as a reference material is its brittleness, since it will fracture readily if dropped on a hard surface. Because of the foregoing, a low-expansion metal (such as Invar or a similar alloy) may offer a preferable alternative if the alloy has repeatable and accurately known expansion properties over the temperature range of interest.

#### Strain Gage Selection

The type of strain gage selected for use in measuring expansion coefficients is also an important consideration, just as it is for stress analysis and transducer applications. Gage selection usually requires weighing a variety of factors which can directly or indirectly affect the suitability of a particular gage type to a specified measurement task. To assist gage users in this process, our Tech Note TN-505 provides extensive background data for gage selection, along with procedures, recommendations, and application examples<sup>6</sup>. The subject Tech Note should serve as the primary reference on gage selection, supplemented here by special considerations applicable to the measurement of expansion coefficients.

For good accuracy, combined with ease of installation, a gage from Vishay Micro-Measurements CEA Series is ordinarily a suitable choice. This assumes that the tempera-

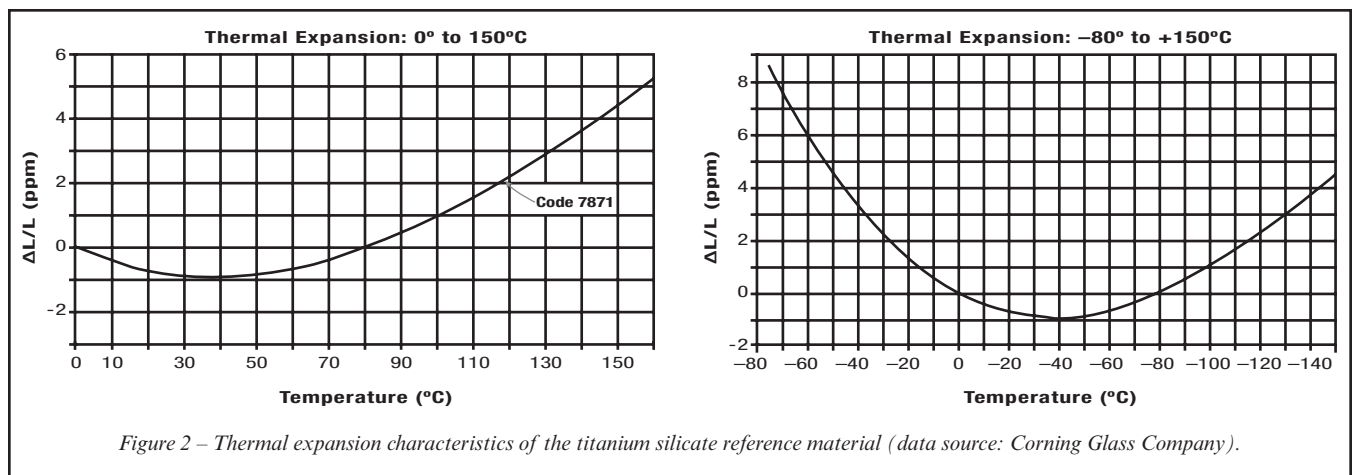


Figure 2 – Thermal expansion characteristics of the titanium silicate reference material (data source: Corning Glass Company).

















